Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 51-1 (2018) 644–649

Normalized Pseudo-Control for Solid Divert & Attitude Control System

Avnish Kumar * **Shashikala Sinha** ** **Abhijit Bhattacharya** ***

* *PGAD/RCI, DRDO, Hyderabad, 500058 INDIA (e-mail: avnish.rect@ gmail.com).*
** *PGAD/RCI, DRDO, Hyderabad, 500058 INDIA.*
, *** *PGAD/RCI, DRDO, Hyderabad, 500058 INDIA.*

Abstract: Faster trajectory correction and control mechanisms are required to correct the radar handing-over error coupled with homing guidance demands for Missile Interceptors. Solid Divert and Attitude Control System (SDACS), which typically have a very low time constant, is one of the most efficient and accurate way to meet these demands. The SDACS has challenges both with respect to the Hot Gas Valve (HGV) technology as well as actuator requirements. The paper proposes a novel method to simplify the controller design and reduce actuator needs for the rotary hot gas valves based SDACS. The proposed method formalizes a linear Pseudo-control to track the lateral acceleration and attitude correction commands. The pseudo-control requirements are tracked through the HGV rotations. The paper further proposes the normalization of the pseudo-control to make the controller generic making it invariant of the valve size and relative dimensions of the HGV's static & moving part. The paper also proposes a technique to reduce the number of actuators required for the full SDACS system by use of these near affine pseudo-controls.

Keywords: Aerospace Engineering, Homing trajectory control, System modelling and identification, Nonlinear systems and control, Pseudo-control, Hot gas valve

NOMENCLATURE

SDACS	Solid Divert & Attitude Control System
HGV	Hot Gas Valve
HTK	Hit-To-Kill
DACS	Divert & Attitude Control System
ADC	Aero-Dynamic Control
KV	Kill Vehicle
RCS	Reaction Control System
CG	Center of Gravity
GG	Gas Generator
EMA	Electro Mechanical Actuator
C_f	Thrust coefficient
F_{Di}	Thrust vector produced by i_{th} Divert nozzle
F_{Ai}	Thrust vector produced by i_{th} Attitude nozzle
F_D	Resultant Divert force vector
F_A	Resultant Attitude force vector
d_i	Moment arm of i_{th} nozzle thrust
M_i	Moment produced by individual Attitude nozzles
F	Thrust produced by a nozzle
P_c	Gas generator chamber pressure
A_t	Throat Area
C_d	Coefficient of Discharge
A	Valve Opening Area
A^*	Normalized Valve Opening Area
A_{max}	Valve Opening Area at Full Open Condition
ϕ	HGV rotation angle
ϕ^*	Normalized HGV rotation angle
ϕ_{max}	HGV minimum rotation angle when in Closed Condition

1. INTRODUCTION

End game kill vehicle of a Hit-To-Kill (HTK) ballistic missile interceptor functions in autonomous mode relying only on onboard sensors. To neutralize the target the kill vehicle is needed to be very agile and should have an accurate and agile control mechanism. Interceptor missiles are generally designed to destroy target either through impact of warhead fragments or by HTK interception, i.e., colliding itself with the target. The higher momentum transfer in the later makes HTK to be preferred choice.

End game kill vehicle of a HTK interceptor functions in autonomous mode relying only on onboard sensors. To home onto the target in faster and accurate way it needs a trajectory correction capability as well as attitude correction capabilities. With these capabilities the interceptor missile is required to correct the radar's handing-over error and meet the homing guidance lateral acceleration and orientation demands.

Divert and Attitude Control System (DACS) is employed in the Kill Vehicle (KV) to fulfil the demand of faster trajectory correction and attitude control. A solid propellant based DACS (SDACS) is a compact and low weight solution. But, this requires state-of-the-art variable thrust solid propellant rocket motor technology. Ostrander (2000) details the design and simulation challenges for a variable thrust pintle motor. A detailed listing of challenging technologies viz. modelling and simulation, pintle motor and nozzle design, control and actuation mechanisms etc. are given in Burroughs (2001). These two works gives an insight to the pintle Hot Gas Valve (HGV) technologies and challenges. Ponzo (2009) demonstrates the test results of a Attitude Control Engine employing pintle type

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2018.05.108

HGV made up of Molybdenum and Rhenium alloy. The described HGV system is handling propellant gas of temperatures 3000 ^{0}F and delivering thrust level less than 30 lbf. Due to the lower temperature gas handling capability the system delivers a smaller specific impulse and thus needs larger propellant mass for a specified correction. The smaller thrust level limits its operation to only Reaction Control System (RCS) for attitude control of smaller Kill Vehicles. The present work proposes a mathematical model to carry out control system design for another type of HGV, i.e., rotational HGV based SDACS. The rotational HGV is relatively a low cost solution and uses conventional materials in the HGV components. It needs lower actuator force than pintle HGV to produce similar variable thrusts. But, the rotational HGV faces the problem of asymmetric flows and non-linear thrust-actuation characteristics which can be solved through the control technique presented in this paper.

Hot Gas Valve (HGV) is the backbone of Solid Propellant based Divert And Attitude Control (DACS) system used for trajectory correction and control of Ballistic Missile Defence Interceptors during the homing guidance phase. To achieve Hit-To-Kill intercept not only the homing sensors needs to be accurate, but also the control device should be very fast with lowest possible time constant. The overall control time constant depends on the bandwidth of the control actuators as well as the time constant of the vehicle control mechanisms. This paper discusses the control problem and proposed solution for a Solid Divert & Attitude Control System which is employed to control the interceptor vehicle through solid propellant based variable thrust device. The SDACS systems is manyfold faster than the Aero-Dynamic Control (ADC) systems and more reliable and of lower weight than a liquid propellent based DACS.

This paper is organized as follows. Section 2 is a brief description of construction of a SDACS system. Section 3 covers the theory of SDACS and HGV and gives the mathematical modelling of rotational HGV and control formulations. Simulation and test results are presented in Section 4. Section 5 gives the concluding remarks.

2. SOLID DACS DESCRIPTION

The Solid DACS shall be capable of delivering zero to its maximum designed divert thrust in omni-directions at any point of time during its operation. In addition it need to produce variable or pulsed attitude control moments (Roll, Pitch and Yaw moments). Four numbers of coplanar orthogonal HGV nozzles functions generally mounted at the KV longitudinal CG works in unison to produce desired resultant thrust vector. Another 6 or 8 numbers of nozzles are needed for the purpose of attitude control and needed to be positioned farthest from KV CG in order to have larger moment arm and in-turn smaller force requirement for the required moment generation.

A schematics of SDACS for divert thrust application is shown in Figure 1. The Gas Generator (GG) produces high temperature and high pressure combustion gases through burning of the solid propellant grain. The hot gas flow is then distributed to the four independent HGV nozzles in order to produce the desired controlled thrust through each of them in order to get desired resultant thrust vector. The HGVs are actuated by independent Electro Mechanical Actuators(EMA), which enable the valves to dynamically operate at the desired openings. The actuators are commanded by a common program which is executed through an on-board computer.

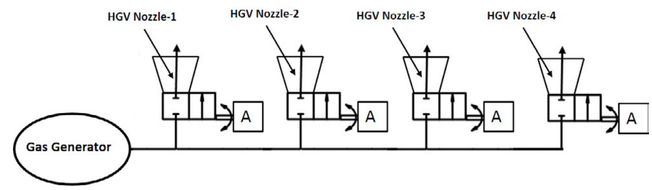

Fig. 1. Schematics of SDACS for Divert Thrust

3. MATHEMATICAL MODEL

The resultant forces and moments produced by the SDACS can be found vectorially from the individual thrust vector component produced by each HGV nozzle. In this section thrust produced by a single HGV nozzle is given.

This paper assumes the flow to be fully developed down to the valve for all the operating positions of HGV. The fully developed flow can be achieved by proper flow path design. [1]

The above assumption makes the thrust coefficients (C_fs) independent of the valve actuation positions. Thus, the thrust is a function of only chamber pressure and valve opening. For the present paper the chamber pressure is assumed to be constant and independent of individual valve opening. [2]

3.1 SDACS Forces & Moments

The forces and moments produced by the SDACS nozzles can be computed by Equations (1) to (3).

$$F_D = \Sigma F_{Di} \tag{1}$$

$$F_A = \Sigma F_{Ai} \tag{2}$$

$$M_i = \Sigma F_i d_i \tag{3}$$

These generalised equations can be used to formulate divert forces for guidance corrections or attitude moments (Pitch, Yaw and Roll) for attitude control of the kill vehicle.

The equation 4 gives the thrust produced by a nozzle Sutton (2001)

$$F = C_f P_c A_t \tag{4}$$

The flow path in the nozzle with HGV will not be smooth like that in an ideal convergent divergent nozzle. After the initial transients the flow becomes fully developed forming an actual throat which may be a little smaller than physical throat (valve opening area). The ratio between the actual throat to physical throat is defined here as

[1] The assumptions holds good for most of the opening positions except when the valve is almost close which may lead to a detached flow.

[2] The invariant of chamber pressure on individual valve opening can be achieved through propellant design and relative valve mechanisation.

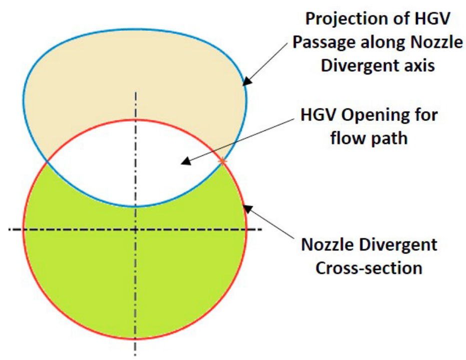

Fig. 2. HGV throat geometry projected along normal to nozzle divergent axis

coefficient of discharge (C_d). Hence, the Equation (4) transforms to Equation (5).

$$F = C_d C_f P_c A_t \tag{5}$$

It has been verified through experiments that $C_d > 0.9$ for majority of the valve opening. In this paper the C_d value is taken as 1. The error induced due to this assumption will contribute towards un-modelled plant dynamics and can be taken care by a proper feedback control design.

Thus for a constant chamber pressure (P_c) and an invariant thrust coefficient (C_f), and by taking C_d as constant equal to unity, the thrust through a nozzle is linearly proportional to throat area.

$$F \propto A_t \tag{6}$$

Hence, by changing the nozzle throat the thrust through that nozzle can be varied proportionally. The subsequent equations will deal with the generation of desired throat area through control of variable throat mechanisms and its consequences.

3.2 Mathematical Modelling of Pseudo-Control

The nozzle is designed to generate thrust along the axis of divergent cone. The rotational valve produces an opening which is of three dimensional geometry. The HGV opening area projected normal to the axis of divergent will be the effective throat area. This projected throat area is shown in Figure 2.

This projected area can be found by numerical integration method or analytically. In this paper, analytical method is followed and the physical projected throat area has been derived. The formula of the throat area has been derived and is given as Equation (7).

$$\begin{aligned} A =\, & r^2(1+\cos\phi)\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{R}{r}\sin\frac{\phi}{2}\right)\right] \\ & - \frac{1}{2}(R^2 - r^2)\sin\phi \log\left[\frac{R\cos\frac{\phi}{2} + \sqrt{r^2 - \left(R\sin\frac{\phi}{2}\right)^2}}{R\cos\frac{\phi}{2} - \sqrt{r^2 - \left(R\sin\frac{\phi}{2}\right)^2}}\right] \end{aligned} \tag{7}$$

Where R is the HGV rotating component radius and r is the valve port radius.

Here HGV valve rotation angle (ϕ) is taken as zero when the valve is fully open condition. By defining $\alpha = \frac{R}{r}$, the Equation (7) can be rewritten as in Equation (8).

$$\begin{aligned} A =\, & r^2(1+\cos\phi)\left[\frac{\pi}{2} - \sin^{-1}\left(\alpha\sin\frac{\phi}{2}\right)\right] - \frac{1}{2}r^2 \\ & \cdot\ (\alpha^2 - 1)\sin\phi \log\left[\frac{\alpha\cos\frac{\phi}{2} + \sqrt{1 - \left(\alpha\sin\frac{\phi}{2}\right)^2}}{\alpha\cos\frac{\phi}{2} - \sqrt{1 - \left(\alpha\sin\frac{\phi}{2}\right)^2}}\right] \end{aligned} \tag{8}$$

From the equation it is quite evident that $|\alpha\sin(\phi/2)|$ should be less than unity. Hence, for the actuation more than this limit, the valve will be fully closed.

$$|\alpha\sin(\phi/2)| \leq 1 \tag{9}$$

$$|\phi| \leq 2\sin^{-1}(\frac{1}{\alpha}) \tag{10}$$

Thus the minimum valve opening angle for the valve to be in fully closed condition is denoted as ϕ_{max} and is given by the Equation (11).

$$\phi_{max} = 2\sin^{-1}(\frac{1}{\alpha}) \tag{11}$$

For example, for R = 8 cm, and r = 4 cm, i.e., $\alpha = 2$, $\phi_{max} = 60^0$. The same can be seen in the plots in Figure 6.

To further generalise the Openings vs. Rotation angle to make relation invariant of the valve design geometry, it is proposed to normalise the opening area with fully open area magnitude and normalise the opening angle with $\phi_m ax$. These normalzed parameters are denoted by A^* & ϕ^* respectively in this paper.

$$A^* = \frac{A}{A_{max}} = \frac{A}{\pi r^2} \tag{12}$$

$$\phi^* = \frac{\phi}{\phi_{max}} = \frac{\phi}{2\sin^{-1}(\frac{1}{\alpha})} \tag{13}$$

The Equation (8) transformes to normalized Equation (14).

$$\begin{aligned} A^* =\, & [1+\cos(\phi^*\phi_{max})] \\ & \cdot \left[\frac{1}{2} - \frac{1}{\pi}\sin^{-1}\left(\alpha\sin\frac{\phi^*\phi_{max}}{2}\right)\right] \\ & - \frac{1}{2\pi}(\alpha^2-1)\sin\phi^*\phi_{max} \\ & \cdot \log\left[\frac{\alpha\cos\frac{\phi^*\phi_{max}}{2} + \sqrt{1 - \left(\alpha\sin\frac{\phi^*\phi_{max}}{2}\right)^2}}{\alpha\cos\frac{\phi^*\phi_{max}}{2} - \sqrt{1 - \left(\alpha\sin\frac{\phi^*\phi_{max}}{2}\right)^2}}\right] \end{aligned} \tag{14}$$

The trajectories for normalized rotation angle against normalized opening area is shown in Figure 7. It can be seen in the figure that the trajectories are nearly invariant on the valve design geometries. In addition it can be seen that the trajectories are affine in most of the portions and becomes nonlinear only when the valve is in near close conditions. As the thrust produced by the nozzle when valve is at near close condition is almost negligible, hence the transformed normalized HGV equations can be approximated as affine without significant deviation from actual behaviour.

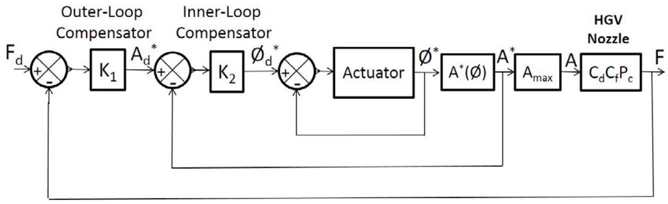

Fig. 3. Three loop controller structure for one SDACS HGV Nozzle

This paper proposes the use of these normalized parameters in the controller design. The proposed method not only makes the non-linear plant model linear but also makes it generic which is invariant of the valve dimensions and hence a generic controller can be designed for a rotational HGV based SDACS system.

The paper proposes the normalized opening areas (A^*s) of SDACS nozzles to be used as pseudo-control parameters to control the forces and moments required by the SDACS. These pseudo-controls are in-turn controlled through the HGV rotations which are driven by Electro-Mechanical Actuators (EMAs) for the typical BMD engagement applications.

3.3 Controller Design Formulation

The force required to be produced through a HGV nozzle is driven through the valve rotation. The plant state equation is written as,

$$\frac{dF}{dt} = \frac{d}{dt}(C_f P_c A_t) \tag{15}$$

By assuming C_f and P_c to be invariant of the instantaneous valve positions,

$$\frac{dF}{dt} = C_f P_c \frac{d}{dt} A_t \tag{16}$$

$$\frac{dF}{dt} = C_f P_c \frac{d}{dt} A_t \tag{17}$$

$$\frac{dF}{dt} = C_f P_c \frac{\partial A_t}{\partial \phi} \frac{d}{dt} \phi \tag{18}$$

The force equation for a HGV nozzle Equation 4 is differentiated with the pseudo-control variable A_t as,

$$\frac{\partial F}{\partial A_t} = C_f P_c. \tag{19}$$

The Equations (12) to (19) are proposed to be used for controller design. A conventional three-loop autopilot can be designed for the purpose. The outermost loop tracks the forces and moments required through normalized pseudo-control (A^*s, which in-turn is tracked through the valve rotation angles (ϕ^*s) in inner-loop while the EMA actuators makes the inner-most loop.

Figure 3 shows the three loop autopilot structure for tracking force generated through a HGV nozzle. The normalized valve opening area (A^*) is the normalized pseudo-control while the normalized valve opening angle (ϕ^*s) is the actual control.

A single actuator can be used to drive two opposite nozzles by making the linkage out of phase. This is possible due to the near-linear nature of pseudo-control - rotation angle characteristics. Similarly another actuator can be used to drive another set of two opposite nozzles.

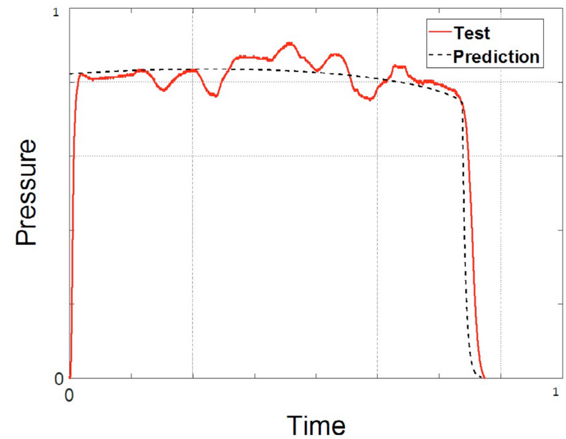

Fig. 4. Test Result: GG chamber pressure - time Plot for SDACS

4. TEST PROGRAM & SIMULATION RESULTS

A SDACS was designed, realized and tested for divert thrust application. The functioning of the Hot Gas Valve was validated through the test. The test was carried with 6-axis load measurement bed. GG chamber pressure was measured during the test.

4.1 Duty Cycle

Elaborate planning for duty cycle has been done for the various tests in order to characterize the system. Initial few tests have been done without actuating the valves. The valves were positioned at predefined opening conditions and fixed. Then test were conducted with dynamic operation of HGV through actuators driven by computer controlled commands. During the operation of rocket motor the HGVs opening area were changed to execute pre-defined duty cycle. Duty cycle were comprised of sine, ramp as well as step command with varied time periods.

4.2 Test Results & inferences

Normalized plots of Pressure vs. Time and Resultant Thrust vs. Time are shown in Figure 4 and Figure 5 respectively for one of the dynamic test case. The test was carried in open loop control mode to assess the capability of designed SDACS and the control challenges it poses. In the test two opposite nozzle valves were kept at equal opening positions and were fixed. The other 2 nozzles, which were opposite to each other were given first ramp and then sine duty cycle. To keep the pressure independent of valve functioning the two valves were operated with 180^0 phase difference.

As outlined in Section 3.1, the predicted pressure trajectory has the basic assumption of an approximation of constant $C_d = 1$ and the deviations due to this assumption was to be characterized through experimental means.

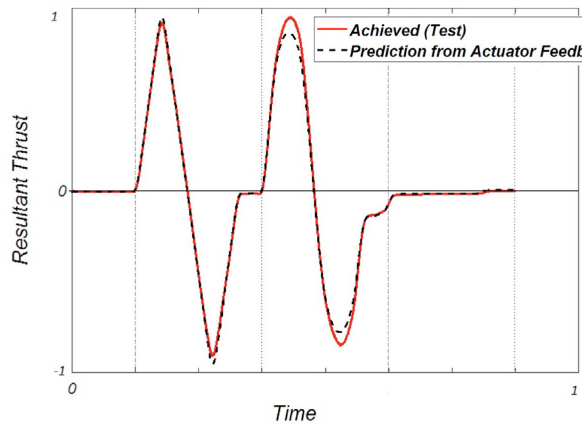

Fig. 5. Test Result: SDACS thrust - time Plot

The test results clearly indicate the nonlinear variation of flow discharge coefficients with valve angular (opening) positions. A number of tests have been carried out to characterize this phenomenon. The variation of discharge coefficients from minuscule opening to full opening is found to be from 0.90 to 0.98. Hence, the significance of the ΔC_d between assumption & test results does not affect the performance greatly and the predictions can be considered as valid.

As can be seen from the thrust profile in the Figure 5 that the output follows the command with negligible time delays. The propulsion time delay (delay between valve actuation and thrust vector) was found in order of less than 10 msec only. This reduces the time constant significantly. Typical ADC time constant is in order of 0.3-0.4 seconds. Major portion of this is due to sluggish body rotation 0.25-0.3 seconds. So, SDACS is able to reduce the vehicle time constant by 60 to 70%. The magnitude of the thrust varies slightly due to variation of discharge coefficient (C_d) with valve opening angles and due to non-linearity as explained in mathematical model as well.

The pressure and thrust trajectories deviations from the designed indicates the non-linear and un-modelled dynamics of the system and from the combined graphs of Figure 4 and Figure 5 the minor deviation of less than 10% from design to test can be seen which is handled through the proposed architecture of controller design.

Hence, through the ground testing of DACS, the strength of HGV based SDCAS to produce a faster trajectory correction control force is demonstrated. Thus the strength of SDACS over ADC is quite evident for an agile application requirement such as interception of incoming missiles.

4.3 Simulation Results

Simulations have been carried out to compute the throat area profile with HGV rotations for various dimensions of HGVs. In the first set of simulations, the valve port radius (r), which corresponds to the maximum valve opening area is kept same while the ratio ($\alpha = R/r$) is varied. r has been taken as 4 cm and the results shown in the Figures 6 & 7. It can be seen that the minimum rotation for closing the HGV decreases as the α increases.

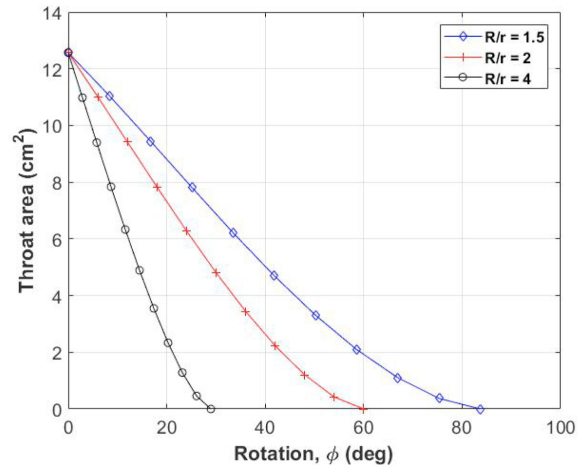

Fig. 6. Plot of Throat Area A_t with HGV rotation angle ϕ for a single HGV

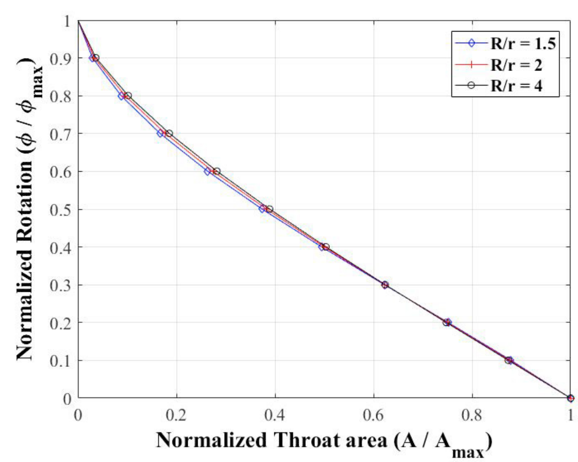

Fig. 7. Plot of Normalized Throat Area A^*,(A/A_{max}) with HGV rotation angle ϕ^*,(ϕ/ϕ_{max}) for a single HGV

In the next set of simulations, both r & α were varied. The results are shown in Figure 8 & Figure 9. Figure 7 & Figure 9 demonstrates the strength of the normalization technique proposed in this paper. From both these plots it is quite evident that the normalized trajectories are almost invariant of the HGV dimensions. It is also clear from the plots that the trajectories are near affine for most of the rotation angles.

Thus, by use of these normalized pseudo-control and normalized rotation angles (control deflection) a generic controller can be designed which will be invariant of the valve dimensions.

5. CONCLUDING REMARKS

The use of SDACS in place of conventional ADC greatly reduces the vehicle time constant. The SDACS vehicle control time delay is mainly due to the sensors and actuators; while the delay contribution due to propulsion of SDCAS is negligible.

This work introduces novel control method for controlling thrust produced through low cost rotary hot gas valve

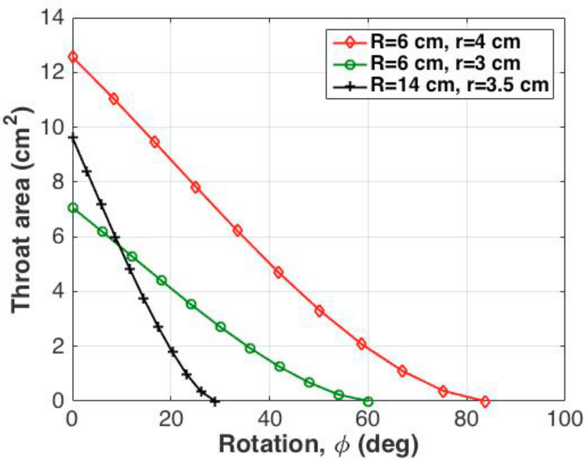

Fig. 8. Plot of Throat Area A_t with HGV rotation angle ϕ for a single HGV

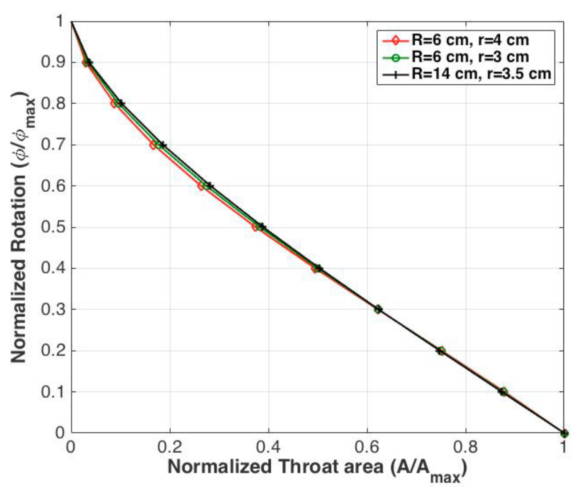

Fig. 9. Plot of Normalized Throat Area A^*,(A/A_{max}) with HGV rotation angle ϕ^*,(ϕ/ϕ_{max}) for a single HGV

controlled rocket motor nozzle. The method simplifies and linearizes non-linear control problem through introduction of a pseudo-control variable as an inner loop track variable. The proposed normalization technique makes the controller design generic, and hence invariant of the valve component dimensions. A scheme to reduce the number of actuators to half is proposed through use of a common actuator to drive two opposite nozzle's HGV through out-of-phase linkages making it cost-effective and more reliable.

It can be concluded that as SDACS has reduced the vehicle time constant, the stringent demand on homing time and hence the seeker tracking range requirement can be relaxed. For an existing seeker it will help to utilize the available homing duration to enhance the capability of the interceptor missile to intercept higher class of targets with lower Radar Cross Section.

ACKNOWLEDGEMENTS

The authors would like to acknowledge J. Ram Mohan, Scientist (Retd.), Advance System Laboratory, Hyderabad for his pioneering effort in the development of SDACS.

REFERENCES

Ostrander, M. and Bergmans, J. and Thomas, M. and Burroughs, S. *Pintle motor challenges for tactical missiles.* 36th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, AIAA-2000-3310, 2000.

Burroughs, Susan. *Status of army pintle technology for controllable thrust propulsion.* 37th Joint Propulsion Conference and Exhibit, AIAA-2001-3598, 2001.

Ponzo, James and Quiggle, Karie and Masse, Robert and Tobin, Sarah. *Long duration hot gas valve demonstration.* 45th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, AIAA-2009-5483, 2009.

Sutton G.P. *Rocket propulsion elements An introduction to the engineering of rockets.* 7th Edition, Wiley & Sons, 2001.